

# Majorana fermions in $s$ -wave noncentrosymmetric superconductor with Rashba and Dresselhaus (110) spin-orbit couplings

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The asymmetric spin-orbit (SO) interactions play a crucial role in realizing topological phases in noncentrosymmetric superconductor (NCS). We investigate the edge states and the vortex core states in  $s$ -wave NCS with Rashba and Dresselhaus (110) SO couplings by both numerical and analytical methods. In particular, we demonstrate that there exists a novel semimetal phase characterized by the flat Andreev bound states in the phase diagram of the  $s$ -wave Dresselhaus NCS which supports the emergence of Majorana fermion (MF). The flat dispersion implies a peak in the density of states which has a clear experimental signature in the tunneling conductance measurements and the MFs proposed here should be experimentally detectable.

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Topological phase of condensed matter systems is the quantum many-body states with nontrivial momentum or real space topology in the Hilbert spaces. A list of the candidate systems includes superfluid  $^3\text{He}$  [1], quantum Hall systems [2, 3], topological insulators (TI) [4–8], topological superconductors (TSC) [9, 10] and so on. Theoretical predictions and experimental observations of TI and TSC have spawned considerable interests recently as a promising tool for spintronics and topological quantum computation (TQC). The TIs are insulators on the inside and conductors on the surface, occurring in the presence of spin-orbit (SO) interactions and time-reversal symmetry (TRS). The SO coupling locks the momentum and spin of the surface electrons and the TRS completely cancels the surface charge current and leads to the pure spin current, as such the TIs open a new gateway for spintronic devices [11–13]. Another peculiar property of the topological phase is that it hosts Majorana fermion (MF) [9, 10, 14, 15] which is its own antiparticle. Discovery of these MFs would be an achievement in itself, and could also lead to a potential breakthrough for technological advance in the fault-tolerant TQC [16] which may overcome the decoherence, the main obstacle to realize a working quantum computer. There are several proposals for supporting MFs in condensed matter systems, for example, chiral  $p$ -wave superconductor [9], superconductor-topological insulator-superconductor (S-TI-S) junction [15], TSC [10, 17, 18] and so on. The TSCs have been first experimentally realized in physical systems of the 1D superconducting indium antimonide nanowire [19] and the Andreev bound states (ABSs) of  $\text{Cu}_x\text{Bi}_2\text{Se}_3$  [20, 21]; the signatures of MFs in S-TI-S structure has also been reported in recent experiment [22]. The S-TI-S junction and TSC could pave the way for the implementation of the TQC and pending to be fully explored theoretically and experimentally.

In this paper, we study the MFs in the edge states and

the vortex core states of a kind of TSC named  $s$ -wave SO coupled noncentrosymmetric superconductor (NCS). Here we focus on the Rashba and Dresselhaus (110) types of SO couplings which are induced by the structure inversion asymmetry and bulk inversion asymmetry, respectively. It is found that the asymmetric SO interaction plays a crucial role in realizing topological phases in the NCS. Although the Rashba SO coupled NCS has been previously investigated [10], the Dresselhaus (110) SO coupled NCS remains unexplored. To obtain a comprehensive understanding of the impact of the asymmetric SO couplings on the topological phases, we mainly explore the Dresselhaus (110) SO coupled NCS and then consider the interplay between them. Interestingly, we find that there is a novel semimetal phase in the Dresselhaus NCS, where the energy gap closes at some points in the first Brillouin zone (BZ) and different kinds of flat ABSs emerge. We demonstrate that these flat ABSs support the emergence of the MFs analytically and numerically. It is known that the Chern number is not a well-defined topological invariant in the gapless region, however, we find that the different kinds of flat ABSs can still be distinguished by the number of gap-closing points in the first BZ. These dispersionless ABSs featured by vanishing group velocity can appear at certain energy, depending on the region of phase diagram in which we choose the parameters. Previously, several authors proposed the flat ABSs in NCS  $\text{Li}_2\text{PdPt}_{3-x}\text{B}$  with high order SO couplings [23, 24],  $d_{xy}$ -wave superconductor,  $p_x$ -wave superconductor and  $d_{xy} + p$ -wave superconductor [25]. Instead we study the flat ABSs in the  $s$ -wave Dresselhaus SO coupled NCS which may be experimentally more feasible. The flat dispersion implies a peak in the density of states (DOS) which is clearly visible and has an experimental signature in the tunneling conductance measurements [21, 23, 26]. The zero-bias conductance peak has been observed in recent experiments on

the indium antimonide nanowire and  $\text{Cu}_x\text{Bi}_2\text{Se}_3$  [19, 20] and argued to be due to the ABS. Thus the flat ABS and the zero-bias conductance peak in the DOS predicted here are experimentally accessible so that the MFs in the Dresselhaus NCS should be detectable.

We begin with modeling the Hamiltonian in a square lattice for the two dimensional  $s$ -wave NCS with Rashba and Dresselhaus (110) SO interactions in a magnetic field, which is given by  $H = H_{\text{kin}} + H_Z + H_R + H_D^{110} + H_s$ :

$$\begin{aligned} H_{\text{kin}} &= -t \sum_{is} \sum_{\hat{\nu}=\hat{x},\hat{y}} (c_{i+\hat{\nu}s}^\dagger c_{is} + c_{i-\hat{\nu}s}^\dagger c_{is}) - \mu \sum_{is} c_{is}^\dagger c_{is}, \\ H_Z &= \sum_{iss'} (V \cdot \sigma)_{ss'} c_{is}^\dagger c_{is'}, \\ H_R &= -\frac{\alpha}{2} \sum_i [(c_{i-\hat{x}\downarrow}^\dagger c_{i\uparrow} - c_{i+\hat{x}\downarrow}^\dagger c_{i\uparrow}) \\ &\quad + i(c_{i-\hat{y}\downarrow}^\dagger c_{i\uparrow} - c_{i+\hat{y}\downarrow}^\dagger c_{i\uparrow}) + \text{H.c.}], \\ H_D^{110} &= -i\frac{\beta}{2} \sum_{iss'} (\sigma_z)_{ss'} (c_{i-\hat{x}s'}^\dagger c_{is} - c_{i+\hat{x}s'}^\dagger c_{is}), \\ H_s &= \sum_i [\Delta_s c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \text{H.c.}], \end{aligned} \quad (1)$$

where  $c_{is}^\dagger (c_{is})$  denotes the creation (annihilation) operator of the electron with spin  $s = (\uparrow, \downarrow)$  at site  $i = (i_x, i_y)$ .  $H_{\text{kin}}$  is the hopping term with hopping amplitude  $t$  and chemical potential  $-\mu$ .  $H_Z$  is the Zeeman field induced by the magnetic field with components  $\mathbf{V} = (V_x, V_y, V_z) = \frac{g\mu_B}{2}(B_x, B_y, B_z)$ .  $H_R$  and  $H_D^{110}$  are the Rashba and Dresselhaus (110) SO couplings and  $H_s$  is the  $s$ -wave superconducting term with gap function  $\Delta_s$ . We assume  $t > 0$  throughout this paper. In the momentum space, the Hamiltonian is  $H = \frac{1}{2} \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger \mathcal{H}(\mathbf{k}) \psi_{\mathbf{k}}$  with  $\psi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{\mathbf{k}\downarrow}^\dagger, c_{-\mathbf{k}\uparrow}, c_{-\mathbf{k}\downarrow})$ , where  $c_{\mathbf{k}s}^\dagger = (1/\sqrt{N}) \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} c_{is}^\dagger$ ,  $\mathbf{k} \in 1\text{BZ}$  and the Bogoliubov-de Gennes (BdG) Hamiltonian

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \xi_{\mathbf{k}} + (\mathcal{L}_{\mathbf{k}} + \mathbf{V}) \cdot \sigma & i\Delta_s \sigma_y \\ -i\Delta_s \sigma_y & -\xi_{\mathbf{k}} + (\mathcal{L}_{\mathbf{k}} - \mathbf{V}) \cdot \sigma^* \end{pmatrix}, \quad (2)$$

where  $\xi_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu$ ,  $\mathcal{L}_{\mathbf{k}} = (\alpha \sin k_y, -\alpha \sin k_x, \beta \sin k_x)$  and  $\sigma = (\sigma_x, \sigma_y, \sigma_z)$  are Pauli matrices.

The nontrivial topological order in the SO coupled NCS is characterized by the existence of gapless chiral edge state and Majorana fermion. The former is determined by the nonzero Chern number and the later can be found at the edge or in the vortex of the system. We shall demonstrate these features in the Hamiltonian Eq. (1) with different parameters. Particularly, we are interested in the following four cases: (i) Rashba NCS in a perpendicular magnetic field,  $\beta = 0$  and  $V_x = V_y = 0$ ; (ii)  $\alpha = \beta$  and  $V_x = V_y = 0$ ; (iii)  $\alpha = \beta$  and  $V_z = 0$ ; (iv)

Dresselhaus NCS in an in-plane magnetic field,  $\alpha = 0$  and  $V_z = 0$ .

To find the topological phases in the Hamiltonian Eq. (2), we can use the Chern number to characterize the nontrivial momentum space topology of the Hamiltonian. The Chern number is defined as  $\mathcal{C} = \frac{1}{2\pi} \int_{T^2} dk_x dk_y \mathcal{F}(\mathbf{k})$ , where  $\mathcal{F}(\mathbf{k}) = \epsilon^{ij} \partial_{k_i} A_j(\mathbf{k})$  is the strength of the gauge field  $A_i(\mathbf{k}) = i \sum_{occ.} \langle \psi_n(\mathbf{k}) | \partial_{k_i} \psi_n(\mathbf{k}) \rangle$ , where  $\psi_n(\mathbf{k})$  is the eigenstates of the Hamiltonian Eq. (2). The integral is carried out in the first Brillouin zone and the summation is carried out for the occupied states of the Hamiltonian Eq. (2). A particular feature of the topological materials is their robustness against perturbations. As long as the topological quantum transition does not happen, the Chern number will remain unchanged. Since the topological quantum transition happens when the energy gap closes, we can depict the phase boundary by studying the gap-closing condition of the Hamiltonian Eq. (2) first. By diagonalizing the BdG Hamiltonian Eq. (2) in the periodic boundary conditions in the  $x$  and  $y$  directions, we can obtain simple analytical solutions for the case (i) and (iv) mentioned in pervious paragraph. The energy spectra of them are  $E(\mathbf{k}) =$

$\pm \sqrt{\xi_{\mathbf{k}}^2 + \mathcal{L}_{\mathbf{k}}^2 + V^2 + \Delta_s^2} \pm 2\sqrt{\xi_{\mathbf{k}}^2 \mathcal{L}_{\mathbf{k}}^2 + V^2(\xi_{\mathbf{k}}^2 + \Delta_s^2)}$ , where  $V = V_z$  for case (i) and  $V = \sqrt{V_x^2 + V_y^2}$  for case (iv). Therefore, we can find that the energy gap will close at  $\xi_{\mathbf{k}}^2 + \mathcal{L}_{\mathbf{k}}^2 + V^2 + \Delta_s^2 = 2\sqrt{\xi_{\mathbf{k}}^2 \mathcal{L}_{\mathbf{k}}^2 + V^2(\xi_{\mathbf{k}}^2 + \Delta_s^2)}$ , which leads to the following gap-closing condition after some straightforward calculations,

$$\xi_{\mathbf{k}}^2 + \Delta_s^2 = V^2, \mathcal{L}_{\mathbf{k}} = 0. \quad (3)$$

For the case (i), the gap closes at  $(k_x, k_y) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$ . By substituting these values into Eq. (3), we obtain three boundary lines,  $(\mu \pm 4t)^2 + \Delta_s^2 = V_z^2$  and  $\mu^2 + \Delta_s^2 = V_z^2$  shown in Fig. (1a). Interestingly, for the case (iv), by the same reason, the gap-closing happens in the line  $k_x = 0$  or  $k_x = \pi$ . Substitute these values into the Eq. (3), we have two equations,  $(\mu \pm 2t + 2t \cos k_y)^2 + \Delta_s^2 = V^2$ , where  $V = \sqrt{V_x^2 + V_y^2}$ . Associating with  $|\cos k_y| \leq 1$ , we can find that the gap closes in the regions from A to G as shown in the Fig. (1b). We also consider the interplay of the Rashba and Dresselhaus (110) SO couplings in the cases (ii) and (iii). There are not simple analytical gap-closing conditions for them. However, as long as the  $z$  component of the magnetic field vanishes, the phase diagram is topologically identical. Therefore, the phase diagram of the case (ii) is topologically the same as Fig. (1a) and the case (iii) is the same as Fig. (1b). In the Fig. (1a), the Chern number is attached for different regions. We can see that in the case (i) and (ii), the corresponding regions have the same Chern number as calculated by M. Sato [10]. Interestingly, different from the case (i) and (ii), in which the gap closes in some boundary lines, in the case (iii)

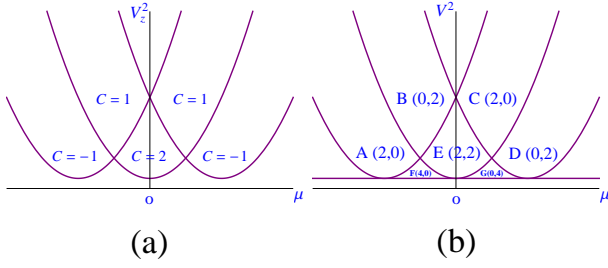


FIG. 1. (color online). The phase diagrams of  $s$ -wave (a) Rashba and (b) Dresselhaus NCS.

and (iv), the gap is closed in the areas from A to G as shown in the Fig. (1b), which means that the system is in the semimetal phase in these regions. Inside the gapless regions, it is well known that the Chern number is not well-defined. In the following, we shall use the gap-closing points in the two lines  $k_x = 0$  and  $k_x = \pi$  to classify the semimetal phase in Fig. (1b). To demonstrate the topological features of the  $s$ -wave Rashba and Dresselhaus SO coupled NCS, we study the ABSs and MFs at the edge and in the vortex core of them. In the first two cases, the phase diagrams are topologically equivalent to the case (i), the Rashba NCS, which is already studied by M. Sato [10, 27]. In the last two cases, the phase diagrams are topologically equivalent to the case (iv), the Dresselhaus NCS. We shall focus on the Dresselhaus NCS in the following paper and see what happens inside the semimetal phase.

We now turn to study the edge states of the Dresselhaus NCS. By setting the boundary condition of  $x$  direction to be open and  $y$  to be periodic, we diagonalize the Hamiltonian Eq. (2) in the cylindrical symmetry and get the edge spectra of the Hamiltonian. The results are depicted in the Fig. (2). Interestingly, although the gap closes in the semimetal phase from A to G as shown in the Fig. (1b), there exist dispersionless ABSs at the edge of the system as shown in Fig. (2). Later we shall demonstrate that these flat ABSs are MFs. The number and position of the edge states depend on the number of the gap-closing points of the Dresselhaus NCS discussed above. From the gap-closing condition, we have  $\cos k_y = \frac{\pm\sqrt{V^2 - \Delta_s^2} - \mu}{2t} \pm 1$ . We can find that different number of solutions to  $k_y$  in the different regions of the phase diagram Fig. (1b). If we define the number of gap-closing points situated in the line  $k_x = 0$  as  $\nu_1$  and in the line  $k_x = \pi$  as  $\nu_2$ , then we can use  $(\nu_1, \nu_2)$  to distinguish different regions in the phase diagram. After some straightforward calculations, we get that (2, 0) for the region A and C, (0, 2) for the region B and D, (2, 2) for the region E, (4, 0) for the region F, (0, 4) for the region G. Consequently, the edge spectrum of region A is topologically equivalent to the region B, C and D, and similar relation for region F to region G. The edge spectra for

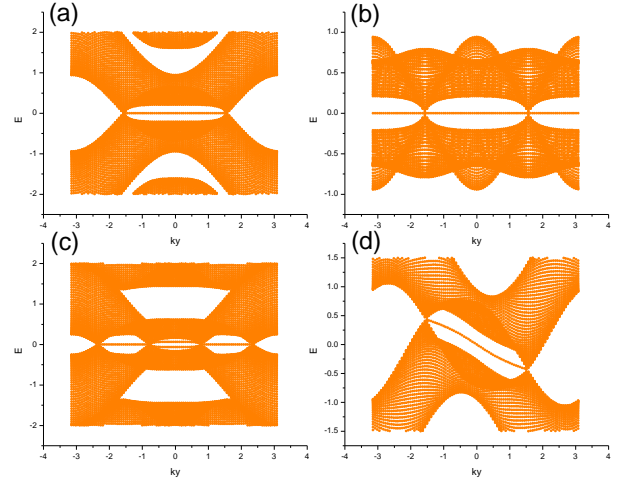


FIG. 2. (color online). The edge spectra of  $s$ -wave NCS with Rashba and Dresselhaus SO interactions. The open edges are at  $i_x = 0$  and  $i_x = 50$ ,  $k_y$  denotes the momentum in the  $y$ -direction and  $k_y \in (-\pi, \pi]$ . For (a)-(c), which correspond to the three regions A, E and F in the Fig. (1b), the parameters are  $t = 1$ ,  $V_z = 0$ ,  $\alpha = 0$ ,  $\beta = 1$ ,  $\Delta_s = 1$ . The chemical potential and in-plane magnetic field are (a)  $\mu = -4$ ,  $V^2 = 5$ , (b)  $\mu = 0$ ,  $V^2 = 5$ , (c)  $\mu = -2$ ,  $V^2 = 3$ . For (d),  $t = 1$ ,  $V_z = 0$ ,  $\alpha = \beta = 1$ ,  $\Delta_s = 1$ ,  $\mu = -4$ ,  $V^2 = 5$ .

the region A to D are shown in the Fig. (2a), for the region E is shown in Fig. (2b) and for the region F and G are shown in Fig. (2c). We observe that there exist different kinds of the zero energy flat ABSs in the different regions of the semimetal phase. Especially, apart from the zero energy flat ABSs, we also find that there are flat ABSs at some nonzero energy in the region F and G. All of these flat ABSs have clear experimental signature in the tunneling conductance measurements, therefore, the MFs emerging in the Dresselhaus NCS should be experimentally observable. We also show the edge spectrum of the case (iii) in the region A and compare with Fig. (2a). Interestingly, from Fig. (2d), we find that the group velocity ( $\partial E / \partial k$ ) of the MFs is tunable by interplaying the Rashba and Dresselhaus SO interactions.

The existence of the edge states implies the nontrivial momentum space topology in the Dresselhaus NCS so that the MFs will emerge at the edge of the system [10]. In the following, we explicitly calculate the Majorana zero modes at the edge of the Dresselhaus NCS in the cylindrical symmetry. Let  $x$  direction to be open boundary and  $y$  to be periodic, then  $k_x \rightarrow -i\partial_x$ , we solve the Schrödinger equation of Hamiltonian Eq. (2) in the real space,  $H(k_x \rightarrow -i\partial_x, k_y)\Psi = 0$ , where  $\Psi = (u_\uparrow, u_\downarrow, v_\uparrow, v_\downarrow)^T$ . Due to the particle-hole symmetry in the Dresselhaus NCS, we have  $u_\uparrow = v_\uparrow^*$  and  $u_\downarrow = v_\downarrow^*$  at zero energy. Thus, we only need to consider the upper block of Hamiltonian Eq. (2). For simplicity, we consider the low energy theory at  $k_x = 0$ , up to the first order, we

have

$$\begin{aligned} (\varepsilon(k_y) - i\beta\partial_x)u_\uparrow + (V_x - iV_y)u_\downarrow + \Delta_s u_\downarrow^* &= 0, \\ (\varepsilon(k_y) + i\beta\partial_x)u_\downarrow + (V_x + iV_y)u_\uparrow - \Delta_s u_\uparrow^* &= 0, \end{aligned} \quad (4)$$

where  $\varepsilon(k_y) = -2t(1 + \cos k_y) - \mu$ . Observe that  $u_\uparrow = \pm iu_\downarrow^*$  in Eq. (4), we have

$$(-i\beta\partial_x + \varepsilon(k_y) \mp i\Delta_s)u_\uparrow \pm (V_y + iV_x)u_\uparrow^* = 0. \quad (5)$$

Solving this equation directly, we obtain that when  $u_\uparrow = iu_\downarrow^*$ , the solution is  $u_\uparrow(x) = c_1 u_\uparrow^1(x) + c_2 u_\uparrow^2(x)$ , where  $c_1$  and  $c_2$  are real numbers and

$$\begin{aligned} u_\uparrow^1(x) &= A_1 e^{\lambda_1 x} + A_2 e^{\lambda_2 x}, \\ u_\uparrow^2(x) &= iB_1 e^{\lambda_1 x} + iB_2 e^{\lambda_2 x}, \end{aligned} \quad (6)$$

where  $\lambda_{1,2} = \frac{-\Delta_s \pm \sqrt{V^2 - \varepsilon^2}}{\beta}$  and  $A_{1,2} = \frac{1}{2}(1 \mp \frac{V_x - i(V_y + \varepsilon)}{\sqrt{V^2 - \varepsilon^2}})$ ,  $B_{1,2} = \frac{1}{2}(1 \pm \frac{V_x - i(V_y - \varepsilon)}{\sqrt{V^2 - \varepsilon^2}})$ ; when  $u_\uparrow = -iu_\downarrow^*$ , the solution is  $u_\uparrow(x) = c_3 u_\uparrow^3(x) + c_4 u_\uparrow^4(x)$ , where  $c_3$  and  $c_4$  are real numbers and

$$\begin{aligned} u_\uparrow^3(x) &= iA_1 e^{\lambda_3 x} + iA_2 e^{\lambda_4 x}, \\ u_\uparrow^4(x) &= B_1 e^{\lambda_3 x} + B_2 e^{\lambda_4 x}, \end{aligned} \quad (7)$$

where  $\lambda_{3,4} = \frac{\Delta_s \pm \sqrt{V^2 - \varepsilon^2}}{\beta}$ . We consider the Dresselhaus NCS in the positive  $x$  plane with the edge located at  $x = 0$ . Then the critical point for existing of the normalizable wavefunctions under this boundary condition is determined by  $V^2 - \varepsilon(k_y)^2 = \Delta_s^2$ , which is consistent with the gap-closing condition  $(\mu + 2t + 2t \cos k_y)^2 + \Delta_s^2 = V^2$  at  $k_x = 0$ . By the same reason, the condition for normalizable wavefunctions is consistent with the gap-closing condition  $(\mu - 2t + 2t \cos k_y)^2 + \Delta_s^2 = V^2$  at  $k_x = \pi$  if we consider the low energy theory at  $k_x = \pi$ . Let's assume  $\Delta_s > 0$  for simplicity, then the Majorana bound state is  $(u_\uparrow, iu_\uparrow^*, u_\downarrow^*, -iu_\downarrow)^T$ , where  $u_\uparrow$  is the solution of Eq. (6).

To further study the MFs in the Dresselhaus NCS, we consider the zero energy vortex core states by solving the BdG equation for the superconducting order parameter of a single vortex  $\Delta(r, \theta) = \Delta \exp(i\theta)$  [28]. To do this, the  $s$ -wave superconducting term in Hamiltonian Eq. (1) is modified to be position-dependent,  $H_s = \sum_i (\Delta e^{i\theta_i} c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \text{H.c.})$ . We numerically solve the Schrödinger equation  $H\Psi = E\Psi$  for the Hamiltonian in Eq. (1), where  $\Psi = (u_\uparrow, u_\downarrow, v_\uparrow, v_\downarrow)^T$ . At zero energy we have  $u_\uparrow = v_\uparrow^*$  and  $u_\downarrow = v_\downarrow^*$  as the particle-hole symmetry in the Dresselhaus NCS, then the Bogoliubov quasiparticle operator,  $\gamma^\dagger(E) = \sum_i (u_{i\uparrow} c_{i\uparrow}^\dagger + u_{i\downarrow} c_{i\downarrow}^\dagger + v_{i\uparrow} c_{i\uparrow} + v_{i\downarrow} c_{i\downarrow})$  becomes Majorana operator  $\gamma^\dagger(0) = \gamma(0)$ . Below we only consider the zero energy vortex core states for discussing the MFs in the vortex core. Let's set the  $x$  and  $y$  directions to be open boundary, then we solve the BdG equations numerically and calculate the density profile of quasiparticle for the zero energy vortex core states. The

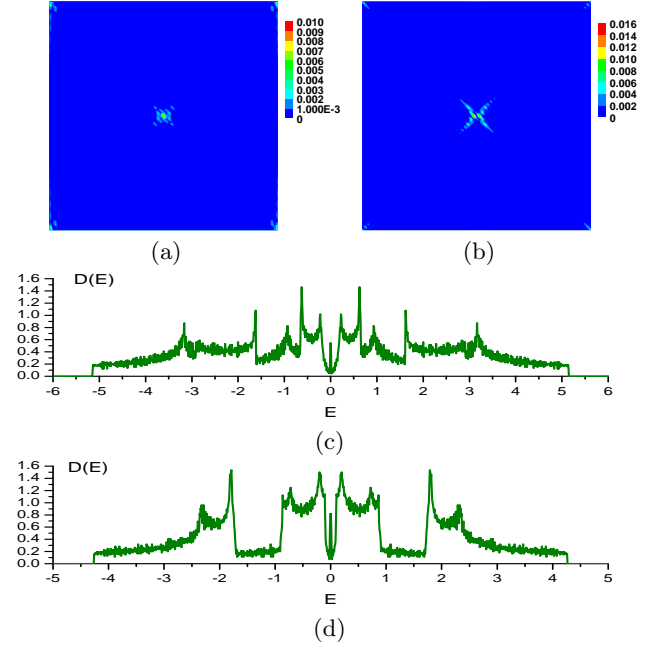


FIG. 3. (color online). The probability distribution and density of states of quasiparticle for the  $s$ -wave Dresselhaus NCS plotted on the  $81 \times 81$  square lattice. The parameters are  $t = 1$ ,  $V_z = 0$ ,  $\alpha = 0$ ,  $\beta = 1$ ,  $\Delta_s = 1$ . The chemical potential and in-plane magnetic field are (a)  $\mu = -4$ ,  $V^2 = 5$  and (b)  $\mu = 0$ ,  $V^2 = 5$ ; (c) and (d) are the density of states for (a) and (b), respectively.

zero energy vortex core states of Rashba NCS has been studied by M. Sato [10, 28]. For fully understanding the impact of the asymmetric SO couplings on the NCS, we explore the Dresselhaus NCS in the present work. Previously, we have shown in Fig. (2) that there is a novel semimetal phase in the Dresselhaus NCS where the zero energy flat edge states host MFs. Here we shall ascertain whether there exist zero energy vortex core states hosting MFs in this semimetal phase. The density profiles of quasiparticle of the zero energy vortex core states are shown in Fig. (3a) and Fig. (3b), which correspond to the region A and E in the phase diagram Fig. (1b), respectively. The numerical results of the energy for the zero energy vortex core states are  $E = 2.27 \times 10^{-4}$  for Fig. (3a) and  $E = 7.11 \times 10^{-6}$  for Fig. (3b), respectively, which are within the simulation tolerance. For the choice of parameters in our simulations, the order of magnitude of  $\Delta_s^2/E_F$  is  $10^{-1}$ . Thus, these numerical results have much smaller energy than the Caroli-de Gennes-Matricon (CdGM) mode [29]. The corresponding DOS are also shown in Fig. (3c) and Fig. (3d). We find that there is a peak at zero energy which further confirms the existence of MFs in the vortex core in the semimetal phase. This peak is measurable in the tunneling conductance experiments and we argue that the MFs predicted in the vortex core may also be detectable.

In summary, we have investigated the topological

phases in  $s$ -wave Rashba and Dresselhaus (110) SO coupled NCS. We study phase diagrams, edge states and vortex core states in different combination of SO couplings. Particularly, we find that there is a novel semimetal phase appearing in the phase diagram of the Dresselhaus NCS. The Chern number is not a well-defined topological invariant for the gapless semimetal phase which however can still be divided into different regions by the number of gap-closing points situated in the line of  $k_x = 0$  and  $k_x = \pi$ . We observe that there exist flat ABSs in the semimetal phase. The flat dispersion leads to a peak at zero energy in the DOS which has a clear experimental signature in the tunneling conductance measurements. We argue that the zero energy flat ABSs at the edge and in the vortex core of the Dresselhaus NCS are MFs by analytical solutions and numerical simulations. Therefore, compare with the proposals for MFs in NCS  $\text{Li}_2\text{Pd}_x\text{Pt}_{3-x}\text{B}$  with high order SO couplings,  $d_{xy}$ -wave superconductor,  $p_x$ -wave superconductor and  $d_{xy} + p$ -wave superconductor, the MFs emerged in the Dresselhaus NCS may be more experimentally accessible.

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- [1] G. E. Volovik, *Exotic Properties of Superfluid  $^3\text{He}$*  (World Scientific, 1992).
- [2] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).
- [3] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).
- [4] B. A. Bernevig and S.-C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006).
- [5] M. König, H. Buhmann, L. W. Molenkamp, T. Hughes, C.-X. Liu, X.-L. Qi, and S.-C. Zhang, Journal of the Physical Society of Japan **77**, 031007 (2008).

- [6] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).
- [7] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 146802 (2005).
- [8] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. **98**, 106803 (2007).
- [9] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
- [10] M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. B **82**, 134521 (2010).
- [11] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007).
- [12] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature **452**, 970 (2008).
- [13] D. Pesin and A. H. MacDonald, Nat. Mat. **11**, 409 (2012).
- [14] W. Bishara, P. Bonderson, C. Nayak, K. Shtengel, and J. K. Slingerland, Phys. Rev. B **80**, 155303 (2009).
- [15] L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008).
- [16] A. Kitaev, Annals of Physics **303**, 2 (2003).
- [17] A. Kitaev, Phys. Usp. **44**, 131 (2001).
- [18] J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, Nat. Phys. **7**, 412 (2011).
- [19] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012).
- [20] S. Sasaki, M. Kriener, K. Segawa, K. Yada, Y. Tanaka, M. Sato, and Y. Ando, Phys. Rev. Lett. **107**, 217001 (2011).
- [21] T. H. Hsieh and L. Fu, Phys. Rev. Lett. **108**, 107005 (2012).
- [22] J. R. Williams, A. J. Bestwick, P. Gallagher, S. S. Hong, Y. Cui, A. S. Bleich, J. G. Analytis, I. R. Fisher, and D. Goldhaber-Gordon, Phys. Rev. Lett. **109**, 056803 (2012).
- [23] A. P. Schnyder and S. Ryu, Phys. Rev. B **84**, 060504 (2011).
- [24] P. M. R. Brydon, A. P. Schnyder, and C. Timm, Phys. Rev. B **84**, 020501 (2011).
- [25] M. Sato, Y. Tanaka, K. Yada, and T. Yokoyama, Phys. Rev. B **83**, 224511 (2011).
- [26] M. Lababidi and E. Zhao, arXiv:1207.5534v1.
- [27] M. Sato, Y. Takahashi, and S. Fujimoto, Phys. Rev. Lett. **103**, 020401 (2009).
- [28] M. Sato and S. Fujimoto, Phys. Rev. B **79**, 094504 (2009).
- [29] C. Caroli, P. D. Gennes, and J. Matricon, Physics Letters **9**, 307 (1964).